2014 EAAC
TAIPEI TAIWAN

18TH EAST ASIAN ACTUARIAL CONFERENCE

12-15 October 2014
Taipei International Convention Center in Taipei Taiwan

Wilkie Stochastic Asset Model

Fitting and Application

By Rakheon Kim

EAAC

Intro – A.D.Wilkie

- Heriot Watt
- Top 3 Actuaries
- Next Top 4?

InQA

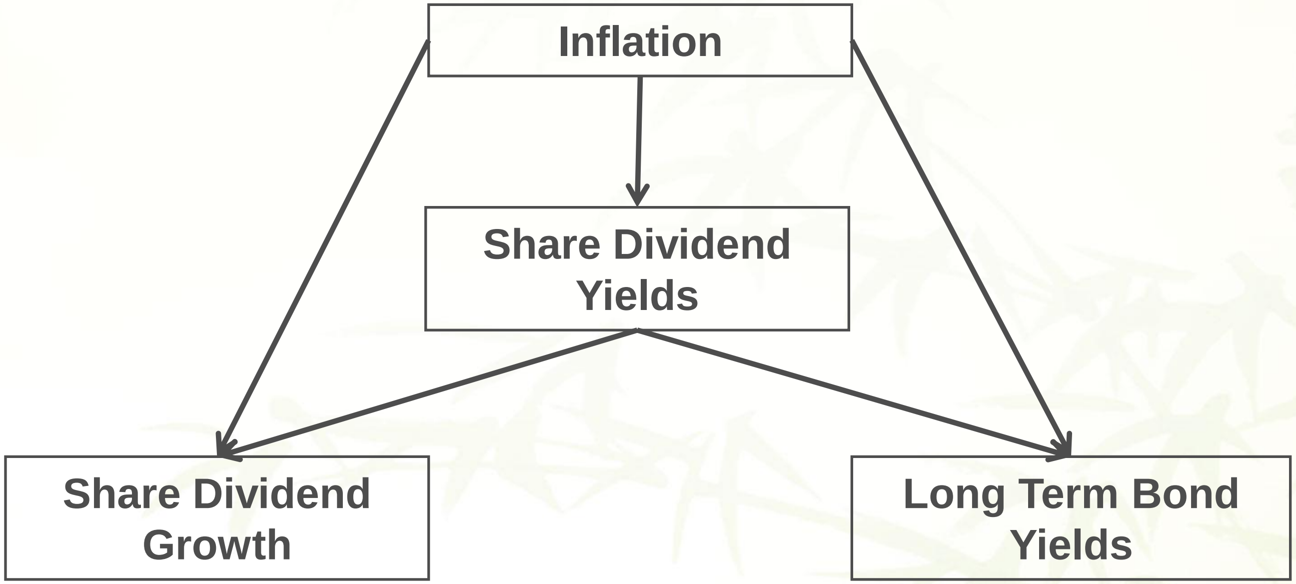
ACTUARIAL SOFTWARE

Contents

- Structure of the Wilkie Model
- Inflation Model
- Share Dividend Yields Model
- Share Dividend Growth Model
- Long Term Bond Yields Model
- Simulation

Structure of the Wilkie Model
"Cascade Structure"

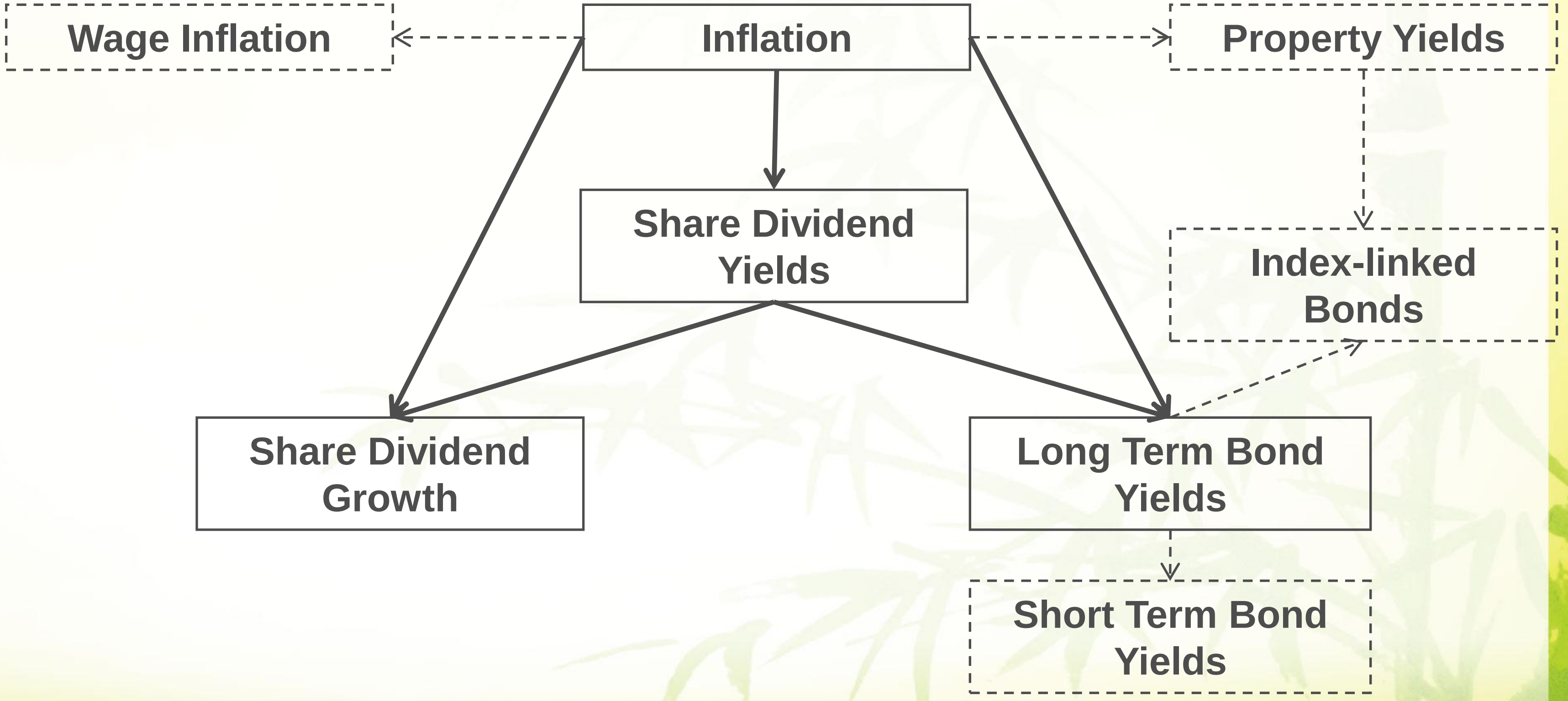
- In the Wilkie model, the interdependence between the economic variables are assumed using "cascade structure"



➡ "Combination of Economics and Statistics"

Structure of the Wilkie Model
"Cascade Structure"

- In the Wilkie model, the interdependence between the economic variables are assumed using "cascade structure"



Inflation Model
Wilkie Inflation Model

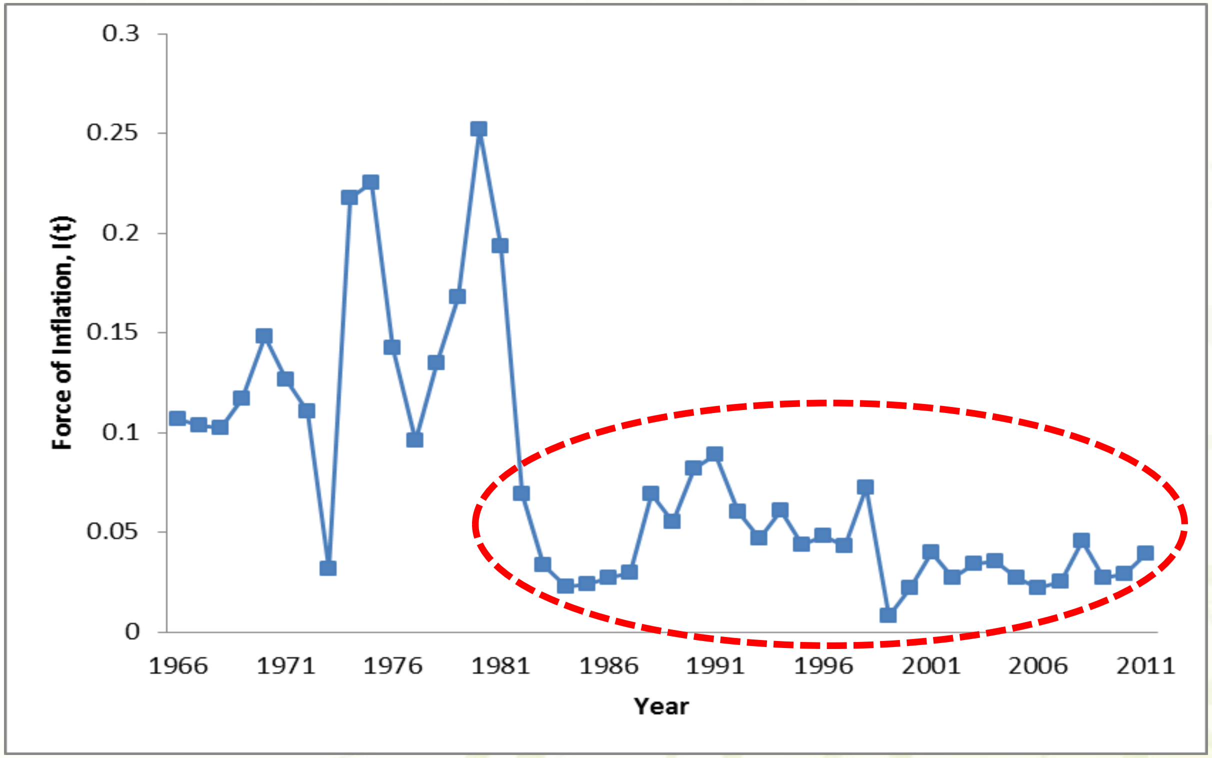
- Wilkie inflation model assumes the annual increment of the price index follows first order autoregressive(AR(1)) process
- Wilkie inflation model
  - Q(t) : retail price index at time t
  - I(t) = ln Q(t) – ln Q(t-1) : inflation rate

$$I(t) = QMU + QA \times \{I(t-1) - QMU\} + QE(t)$$
$$QE(t) = QSD \times QZ(t)$$
$$QZ(t) \sim i.i.d.N(0,1)$$

➡ Tends to depend on the previous level of Inflation
Has autoregressive feature to long term mean inflation (QMU)

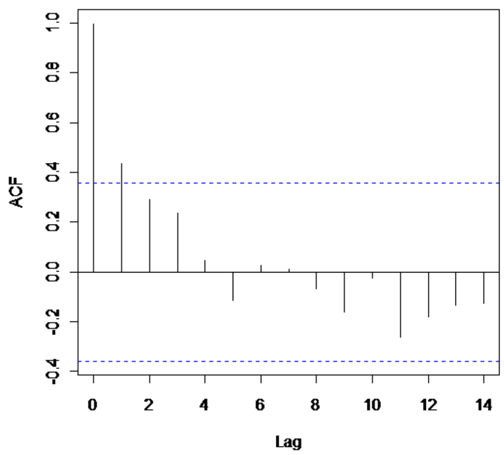
Inflation Model
Application to Korean Market

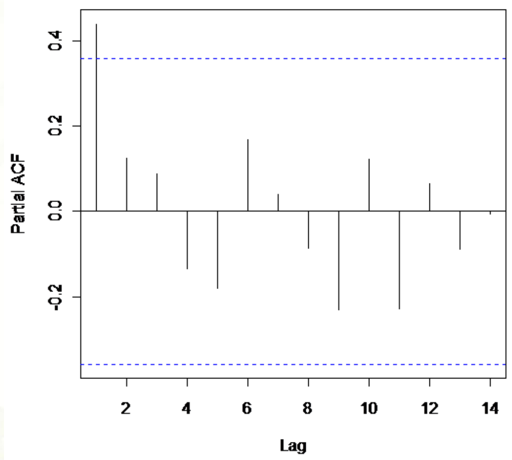
- Data Selection
  - Inflation, I(t), data of the recent 30 years after 1982 chosen



Inflation Model
Application to Korean Market

- Model Fitting
  - auto correlation function & partial auto correlation function → AR(1)





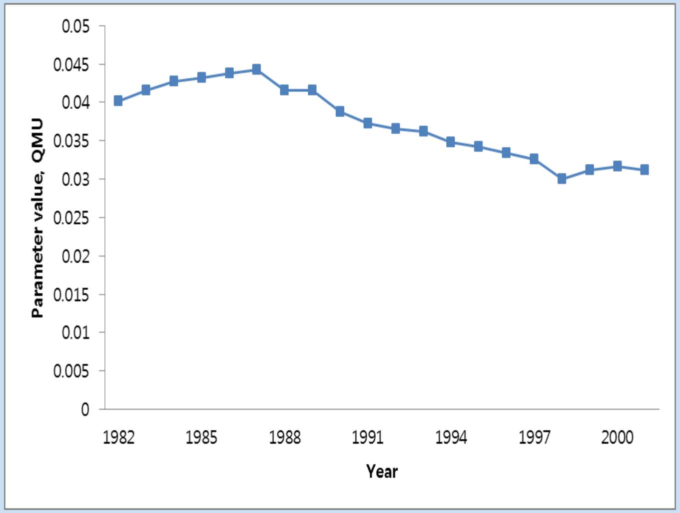
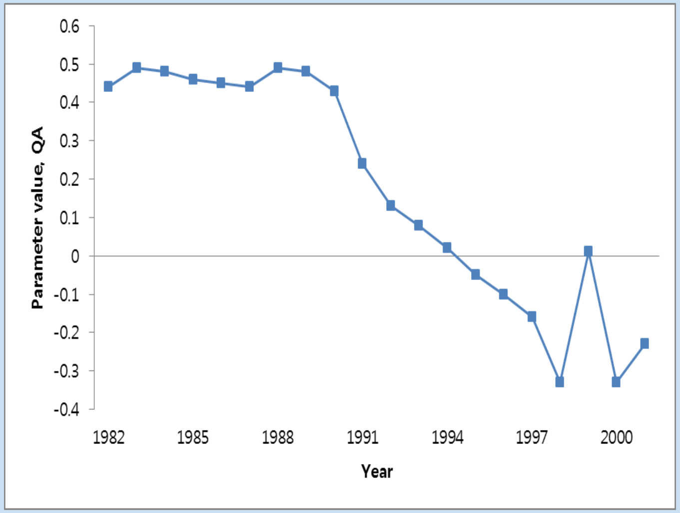
  - Fitted Parameters of the Model

QMU	QA	QSD
0.04	0.44	0.017

Rakheon Kim

Inflation Model
Application to Korean Market

- Model Stability
  - Stability of the Fitted Parameters

QMU	QA
	

Rakheon Kim

Share Dividend Yields Model
Wilkie Share Dividend Yields Model

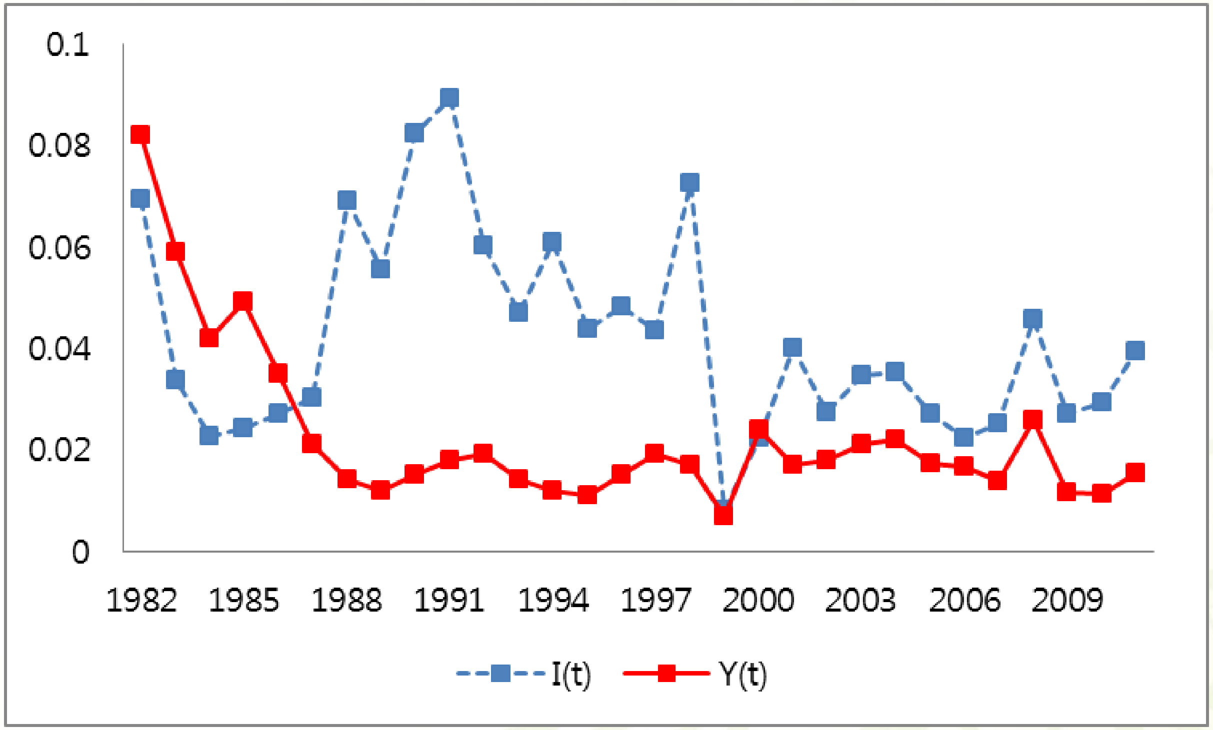
- In the Wilkie model, Dividend Growth Model is used to evaluate the share price
- Wilkie share dividend yields model
  - Y(t) : share dividend yields at time t

$$\ln Y(t) = YW \times I(t) + \ln YMU + YN(t)$$
$$YN(t) = YA \times YN(t-1) + YE(t)$$
$$YE(t) = YSD \times YZ(t)$$
$$YZ(t) \sim i.i.d.N(0,1)$$

➡ Tends to depend on the previous level of dividend yields
Positive correlation with Inflation

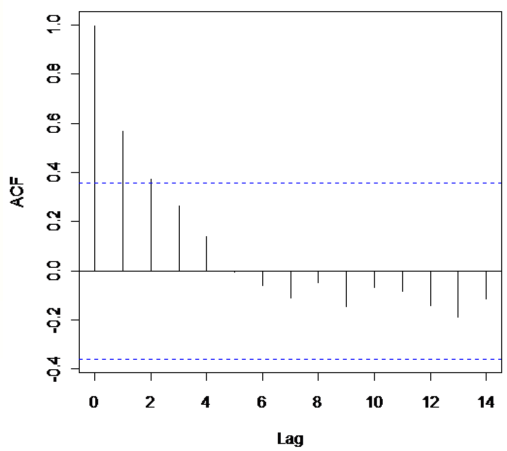
Share Dividend Yields Model
Application to Korean Market

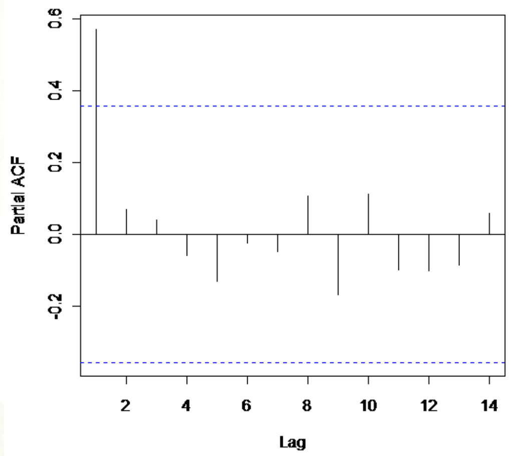
- Data Selection
  - Dividend yields, Y(t), data of the recent 30 years after 1982 chosen



Share Dividend Yields Model
Application to Korean Market

- Model Fitting
  - ACF & PACF of the residuals between lnY(t) and I(t) → AR(1)



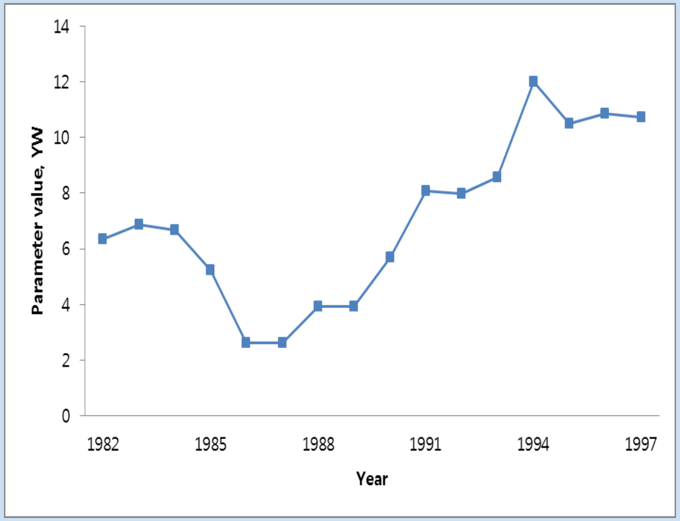
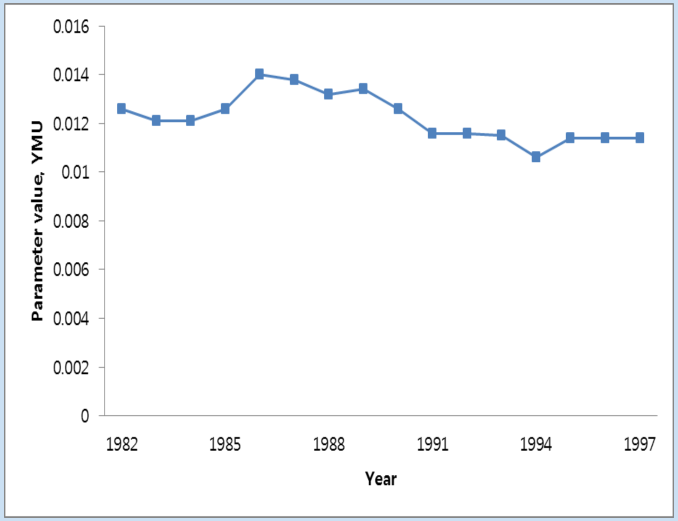


  - Fitted Parameters of the Model

YW	YMU	YA	YSD
6.36	0.013	0.68	0.328

Share Dividend Yields Model
Application to Korean Market

- Model Stability
  - Stability of the Fitted Parameters

YW	YMU
	

Rakheon Kim

Share Dividend Growth Model
Wilkie Share Dividend Growth Model

- In the Wilkie model, Dividend Growth Model is used to evaluate the share price
- The dividend is determined by the following relation
  - K(t) : dividend growth from time t-1 to time t
  - D(t) : dividend amount at time t

$$D(t) = D(t-1) \times \exp\{K(t)\}$$

- Wilkie model derives the share price, P(t), using the Y(t) derived from the dividend yields model and the dividend amount, D(t)
  - P(t) : share price at time t

$$D(t) = P(t) \times Y(t)$$

Share Dividend Growth Model
Wilkie Share Dividend Growth Model

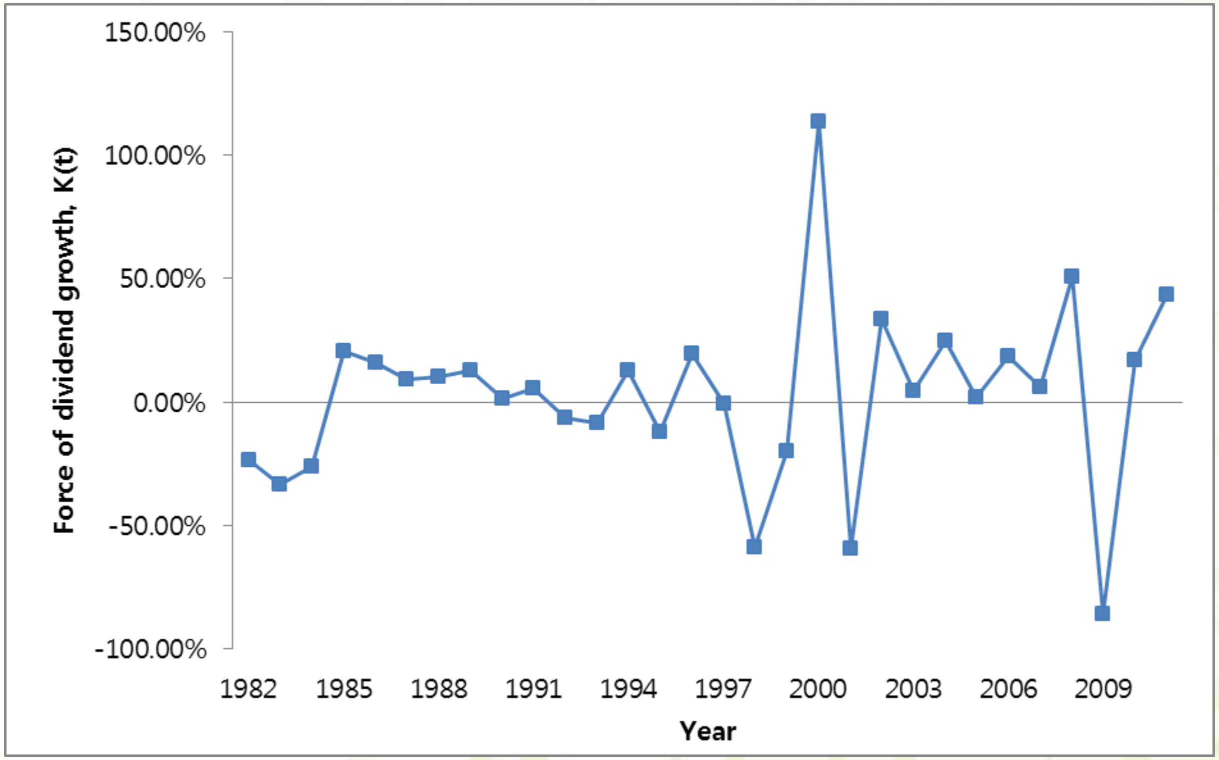
- Wilkie share dividend growth model

$$DM(t) = DD \times I(t) + (1 - DD) \times DM(t-1)$$
$$DI(t) = DW \times DM(t) + (1 - DW) \times I(t)$$
$$K(t) = DI(t) + DMU + DY \times YE(t-1) + DB \times DE(t-1) + DE(t)$$
$$DE(t) = DSD \times DZ(t)$$
$$DZ(t) \sim i.i.d.N(0,1)$$

Depends on inflation and the previous level of dividend yields
- Inflation : Previous level of inflation included as well
- Negative correlation with the previous dividend yields
- Moving average term, DE(t-1) reflects the deferment of profit distribution

Share Dividend Growth Model
Application to Korean Market

- Data Selection
  - Dividend growth, K(t), data of the recent 30 years after 1982 chosen



Share Dividend Growth Model
Application to Korean Market

- Model Fitting
  - Revised model (after fitting DD and DW)

$$K(t) - I(t) = DMU + DY \times YE(t-1) + DB \times DE(t-1) + DE(t)$$
$$DE(t) = DSD \times DZ(t)$$
$$DZ(t) \sim i.i.d.N(0,1)$$

➡ K(t) – I(t) represent the real dividend growth

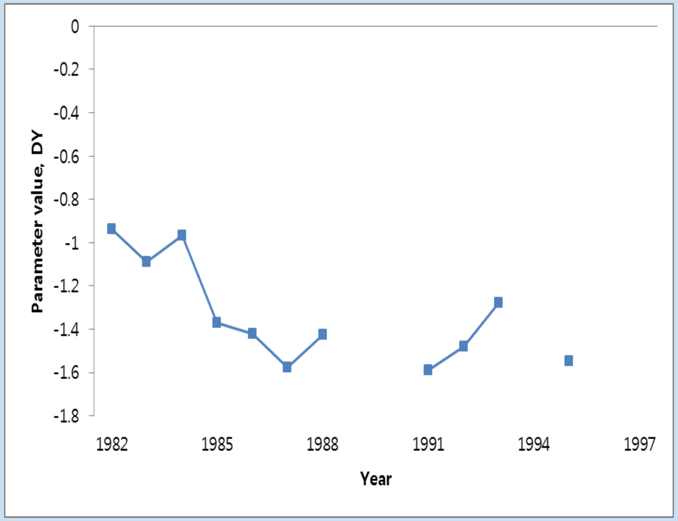
  - Fitted Parameters of the Model

DMU	DY	DB	DSD
0	- 0.94	0.46	0.289

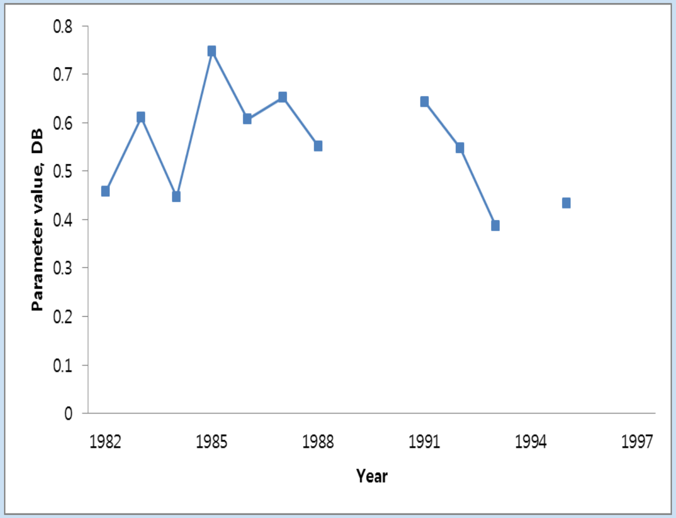
Share Dividend Growth Model
Application to Korean Market

- Model Stability
  - Stability of the Fitted Parameters

DY



DB



Long Term Bond Yields Model
Wilkie Long Term Bond Yields Model

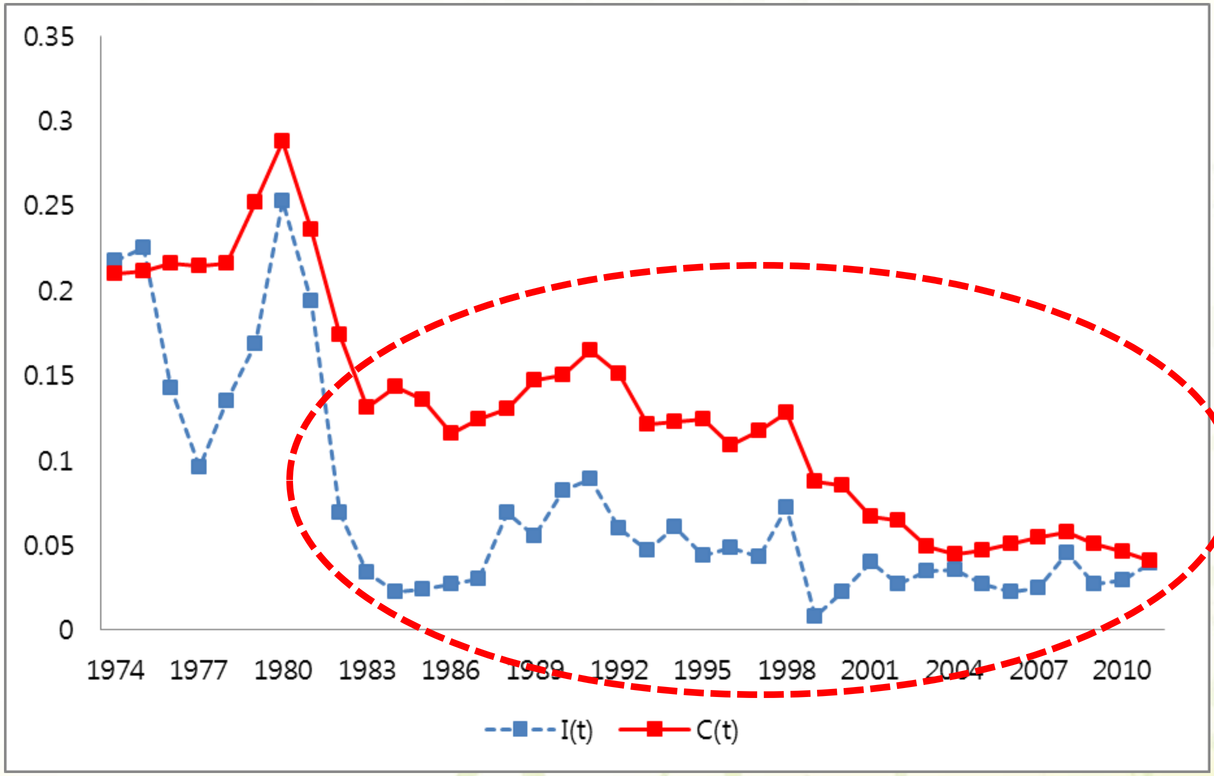
- Wilkie long term bond yields model assumes the 'fisher relation' among nominal yields, real yields and expected future inflation
- Wilkie Long Term Bond Yields Model
  - C(t) : nominal yields, CR(t) : real yields at time t

$$CM(t) = CD \times I(t) + (1 - CD) \times CM(t-1)$$
$$CR(t) = C(t) - CW \times CM(t)$$
$$\ln CR(t) = \ln CMU + CN(t)$$
$$CN(t) = CA \times CN(t-1) + CY \times YE(t) + CE(t)$$
$$CE(t) = CSD \times CZ(t)$$
$$CZ(t) \sim i.i.d.N(0,1)$$

➡ Future inflation expectation based on adaptive expectation
Positive correlation between dividend yields and bond yields

Long Term Bond Yields Model
Application to Korean Market

- Data Selection
  - 10-yr gov't bond yields, C(t), of the recent 30 years after 1982 chosen



Rakheon Kim

Long Term Bond Yields Model
Application to Korean Market

- Model Fitting
  - Revised model for CM(t)

$$CM(t) = CD \times I(t) + \frac{1}{10} \times (1 - CD) \times \sum_{k=1}^{10} I(t-k)$$

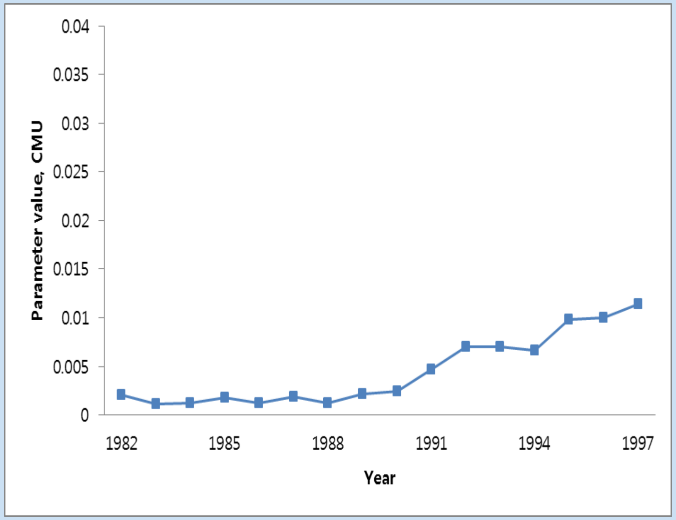
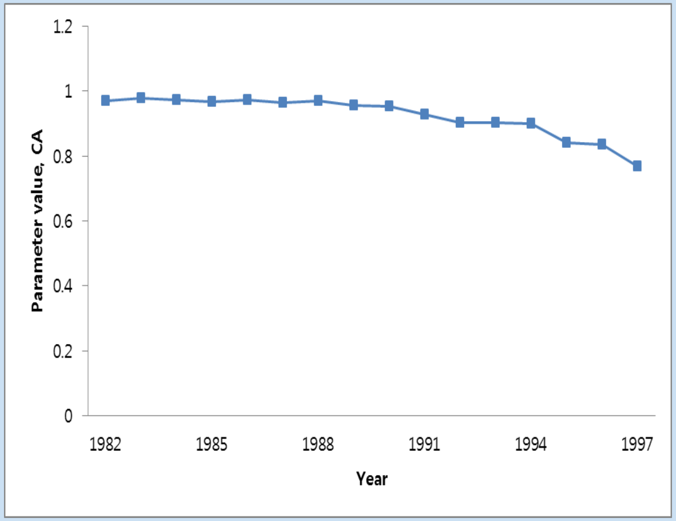
➡ Structural change of the financial market system of the South Korea reflected

- Fitted Parameters of the Model

CMU	CA	CY	CSD
0.002	0.97	0.0	0.34

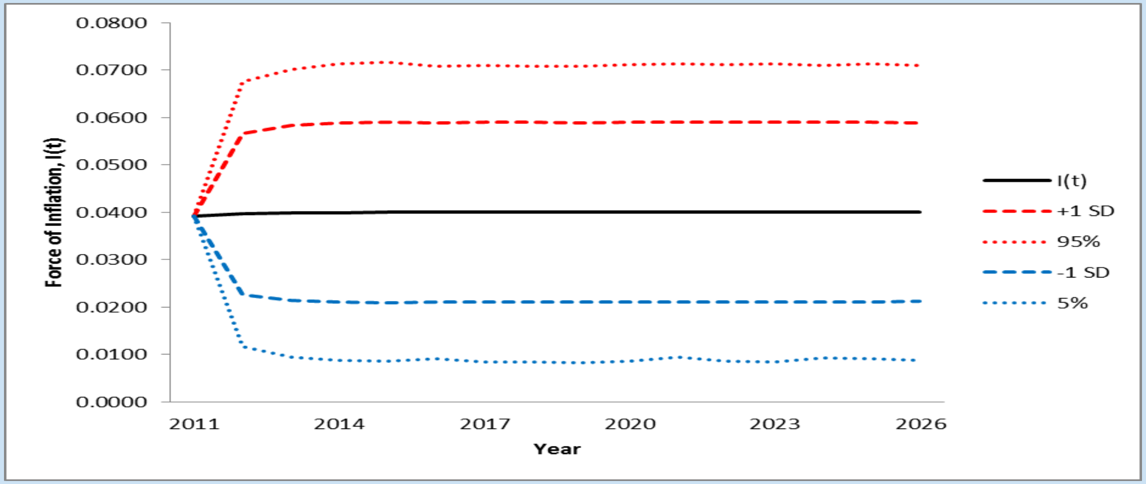
Long Term Bond Yields Model
Application to Korean Market

- Model Stability
  - Stability of the Fitted Parameters

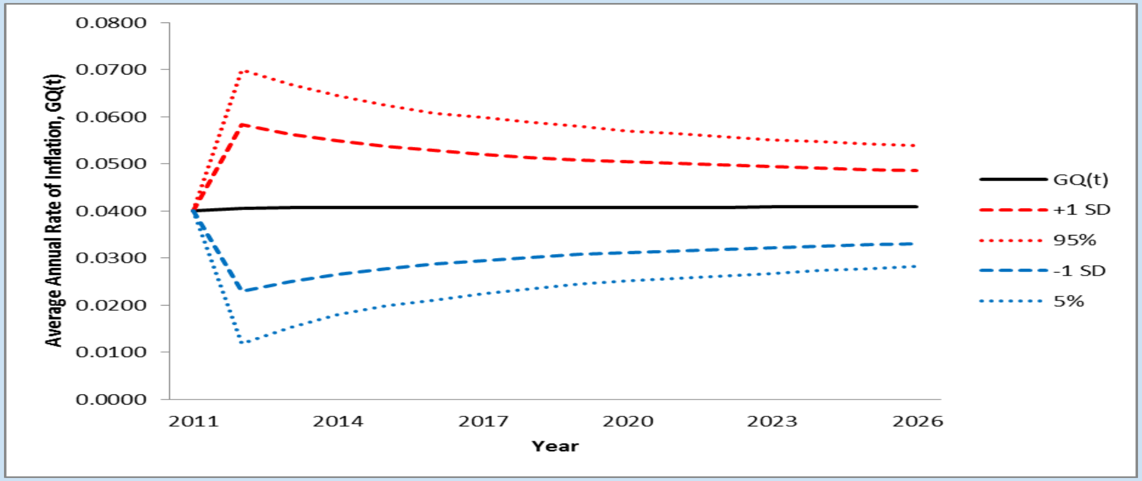
CMU	CA
	

Simulation
Inflation

I(t)



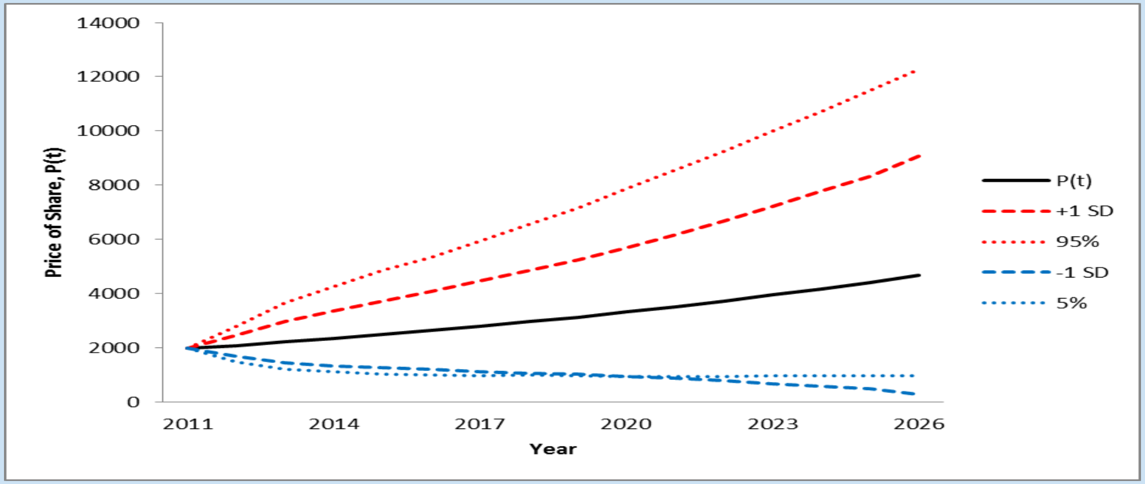
GQ(t)



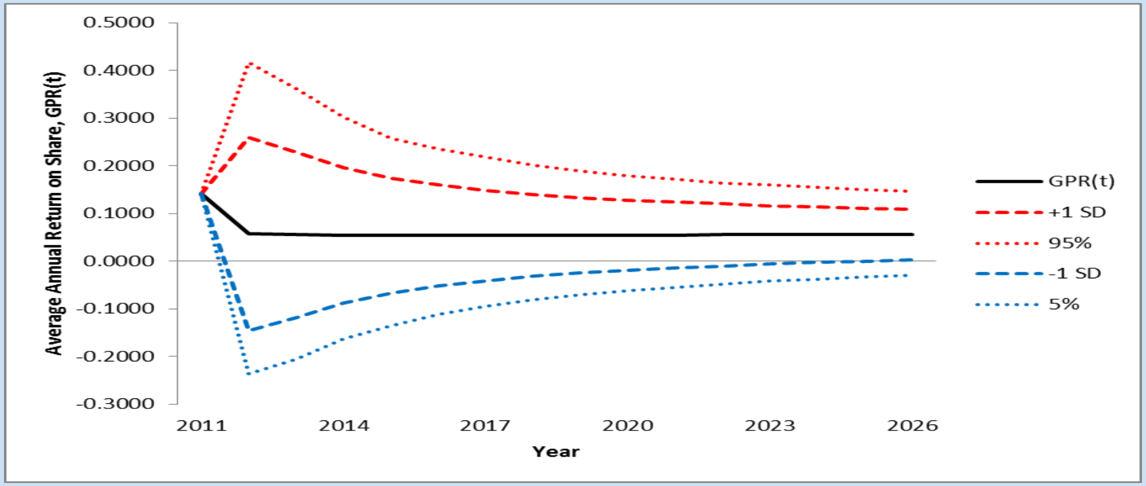
Rakheon Kim

Simulation
Share Price

P(t)



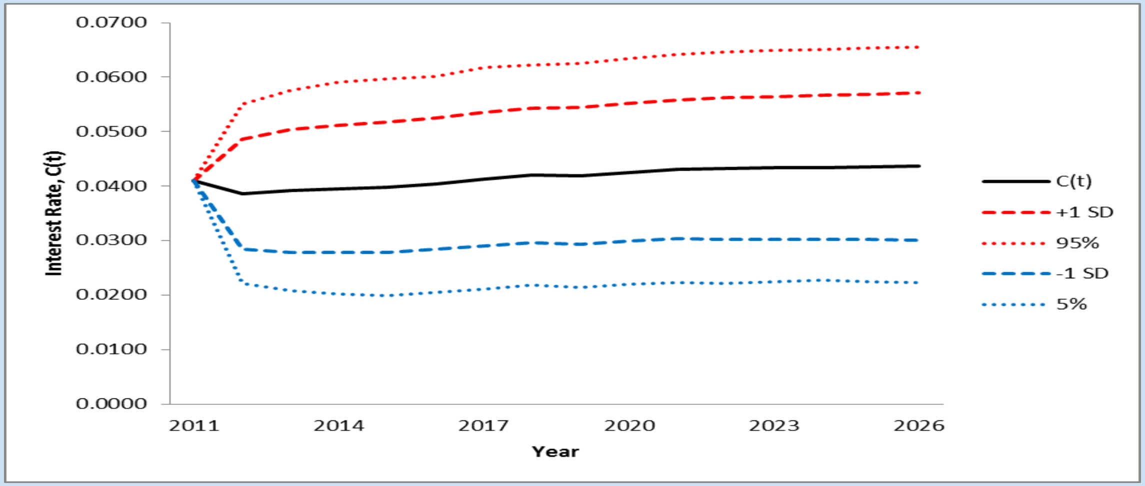
GPR(t)



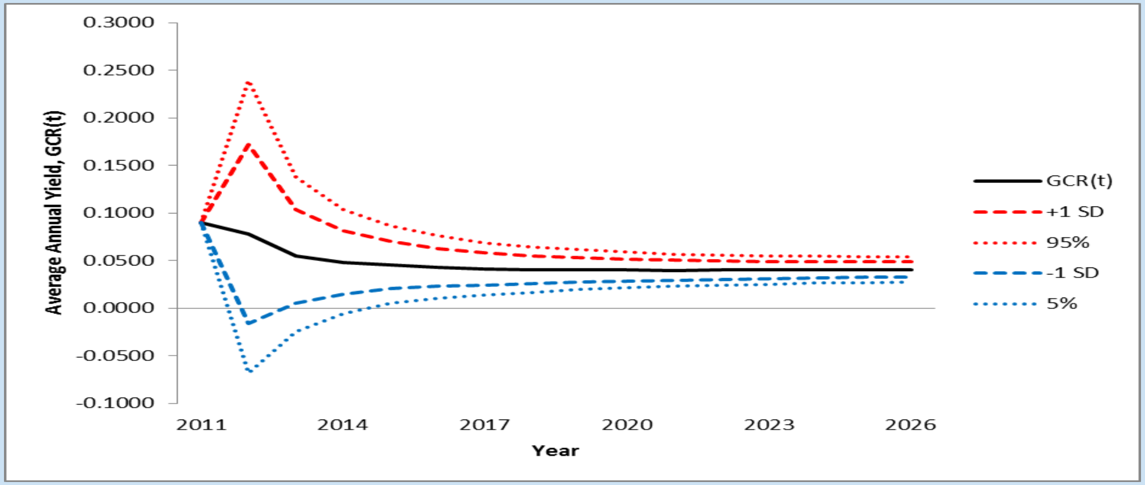
Rakheon Kim

Simulation
Long Term Bond Yields

C(t)



GCR(t)



Rakheon Kim

Thank You